$$B_2 = \frac{\pi}{\nu - \mu} F(d)$$

$$B = \frac{2}{\nu - \mu} \left\{ F(a) \left[K'\bar{Z}(a) + \frac{\pi a}{2K} \right] + F(d) \left[K'Z(d) + \frac{\pi d}{2K} \right] \right\}$$

$$S = \frac{4K}{\nu - \mu} \{ F(a)\bar{Z}(a) + F(d)Z(d) \}$$

$$T = \frac{2}{\nu - \mu} \left\{ F(a) \left[K'\bar{Z}(a) + \frac{\pi}{2}\left(\frac{a}{K} - 1\right) \right] + F(d) \left[K'Z(d) + \frac{\pi}{2}\left(\frac{d}{K} - 1\right) \right] \right\}. \quad (10)$$

THE ODD-MODE FRINGING CAPACITANCE

The odd-mode fringing capacitance of the structure is defined as the limiting value of the difference between the total capacitance of the structure, measured to the magnetic walls, A and D, and the parallel-plate capacitance measured between $0A$ and ED, as the magnetic walls tend to infinity. We thus require the total capacitance of the structure in the z plane out to the magnetic walls at A and D. As we have seen, this figure maps into the upper half t plane so that the magnetic walls at A and D transform into semicircles centered about the points μ and ν, respectively. It is convenient to denote the radii of these circles by $\delta\mu$ and $\delta\nu$ as they approach zero. The capacitance of the structure in the t plane in which one conductor is the line segment between $\mu + \delta\mu$ and $\nu - \delta\nu$ and the other is the infinite line segment between $\nu + \delta\nu$ and $\mu - \delta\mu$ is required, subject to the additional condition that the lines of force are constrained so that the semicircular lines about the endpoints of these line segments are magnetic walls. Riblet [3] has shown recently that the limiting value of this capacitance differs from that of the same structure, in which the endpoints of the line segments are joined by magnetic walls which fall on the real axis, by an amount that is expressible in terms of an excess capacitance, $C_{ex} = \log(2)/\pi$. The capacitance of the structure in the t plane which falls entirely on the real axis is given by $K'(k_0)/K(k_0)$, where

$$k_0^2 = (b - a)\ (d - c)/(d - b)\ (c - a)$$

and

$$a = \mu - \delta\mu, \qquad b = \mu + \delta\mu, \qquad c = \nu - \delta\nu$$

and

$$d = \nu + \delta\nu.$$

Thus

$$k_0^2 = 4\delta\mu\delta\nu/(\nu - \mu)^2$$

in the limit as $\delta\mu$ and $\delta\nu \to 0$. Now

$$\frac{K'(k_0)}{K(k_0)} \approx \frac{1}{\pi} \log \frac{16}{k_0^2} = \frac{1}{\pi} \log \frac{4(\nu - \mu)^2}{\delta\mu\delta\nu}. \quad (11)$$

This capacitance exceeds that of the actual structure by $2C_{ex}$ since there are two vanishing semicircles. If C_0 is the total capacitance of the structure in Fig. 1, then

$$C_0 = \frac{K'(k_0)}{K(k_0)} - \frac{2 \log (2)}{\pi} = \frac{1}{\pi} \log \frac{(\nu - \mu^2)}{\delta\mu\delta\nu}. \quad (12)$$

The determination of the parallel-plate capacitances, C_{P_A} and C_{P_D} associated with the plate gaps B_1 and B_2, respectively, proceeds in a strainghtforward, purely formal manner from (6) and will not be given here in the interest of brevity. If we call the odd-mode fringing capacitance for the asymmetrical case C_{f0}'', it is given by the limiting value of $C_0 - C_{P_A} - C_{P_D}$. By substituting one finally obtains

$$C_{f0}'' = \frac{2}{\pi} \left\{ a\bar{Z}(a) + dZ(d) + \log H'(0) - \tfrac{1}{2} \log (H(2a)H(2d)) + \log [(1 - k^2 \operatorname{sn}^2 a \operatorname{sn}^2 d)/2k] - \tfrac{1}{2} \log (\operatorname{sn} a \operatorname{cn} a \operatorname{dn} a \operatorname{sn} d \operatorname{cn} d \operatorname{dn} d) + \frac{F(d)}{F(a)} aZ(d) + \frac{F(a)}{F(d)} d\bar{Z}(a) + \frac{1}{2}\left(\frac{F(d)}{F(a)} + \frac{F(a)}{F(d)}\right) \log \frac{\theta(a + d)}{\theta(a - d)} \right\} \quad (13)$$

where $H(x)$ and $\theta(x)$ are the familiar Jacobi theta-functions.

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Transmission-Line Transformation Between Arbitrary Impedances

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***Abstract*—An analytical method for transforming between two complex impedances using a single transmission-line matching section is described.**

In a recent letter, Day [1] presents a graphical method to find an impedance transformer using a single transmission-line section. This can also be done analytically using the following formula to transform:

$$Z_1 = R_1 + jX_1$$

to

$$Z_2 = R_2 + jX_2.$$

The transforming line impedance is given by

$$Z_L = \left(\frac{R_1 \cdot |Z_2|^2 - R_2 \cdot |Z_1|^2}{R_2 - R_1} \right)^{1/2}.$$

The transforming line length is given by

$$B = \tan^{-1} \left(\frac{Z_L \cdot (R_2 - R_1)}{R_2 \cdot X_1 + R_1 \cdot X_2} \right)$$

in degrees (or radians).

If B is negative, add 180 (Pi) to get proper length. If the transformation is not possible, Z_L^2 will be negative.

The method can be easily applied on a hand calculator or computer and proves to be much faster and more accurate than a graphical technique.

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